How Haylock and Cockburn and the connective model have shaped and inspired our thinking for 25 years

Ruth Trundley, Andy Tynemouth, Helen Edginton, and Stefanie Burke share their version of the connective model.

In 1989 Derek Haylock and Anne Cockburn published *Understanding Early Years Mathematics,* which later became *Understanding Mathematics in the Lower Primary Years* (1997). These were books we encountered in the late 1990s and early 2000s as a team of mathematics advisers in Devon. We were particularly struck by, and interested in, the model introduced as 'A model of understanding: making connections' (Figure 1) :

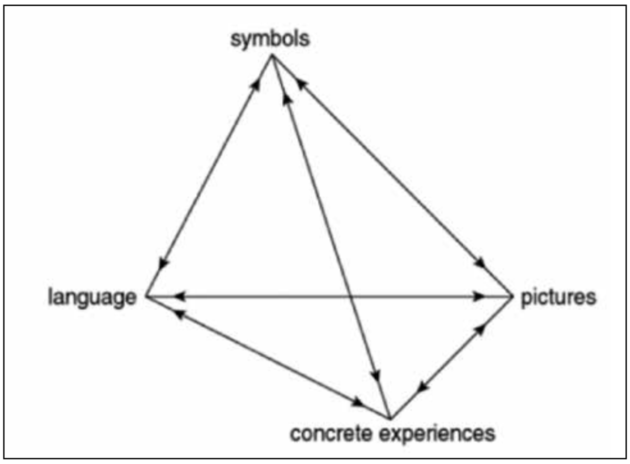


Figure 1: Haylock and Cockburn 1989.

We noticed that the model included Bruner's (1964) three modes of representation (enactive, iconic, symbolic) that have been translated into the CPA approach (concrete, pictorial, abstract). We felt the model offered more than the CPA approach for two main reasons:

- the inclusion of language as a fourth representation, which is consonant with Bruner's belief in social constructivism and Vygotsky's emphasis on the role of language in learning.
- the bi-directional arrows between each pair of representations making it clear that movement between representations should be multi-directional. This movement is reflected in the aims of the national curriculum (2013): "Mathematics is an interconnected subject in which pupils need to be able to move fluently between representations of mathematical ideas" (p. 3) and contrasts with the widely held misapprehension that CPA means that learners only ever go in one direction from concrete to pictorial to abstract.

Haylock and Cockburn suggested that when learners engage in mathematical activity, it will involve manipulating some, or all, of the elements of the model and that connecting these experiences is how learners make sense of the mathematics; hence we refer to it as 'the connective model'. A focus on making sense of the mathematics, rather than a focus on answer getting, also appealed to us, fitting with our beliefs and values about mathematics teaching and learning and the importance of conceptual understanding, and we became hooked.

The arrows in the model indicate learners noticing and articulating their thinking about connections and talk is embedded in these arrows. Consider learners thinking about the composition of six. They have just been to the local park, which has a slide and a climbing frame, and they are now exploring six children in the park playing on either the slide or the climbing frame. They can use small world people, cubes, or counters to represent the six children and are moving them around to make different combinations (Figure 2). They record each of their combinations with a drawing.

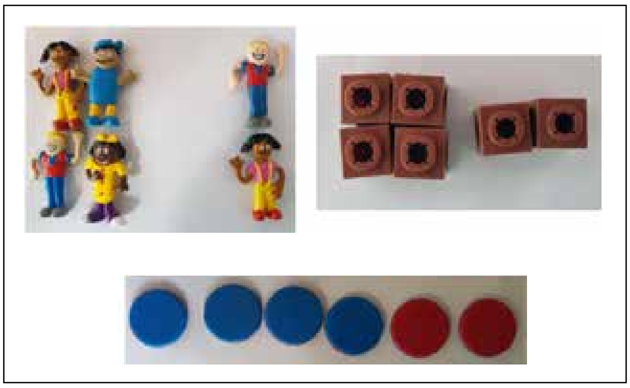


Figure 2: Representations of six children.

Discussion about the representations and combinations includes the learners explaining, "There are six children, four are playing on the climbing frame and two are playing on the slide" and showing where these children are represented in the resources and the drawings using gesture. The adult draws attention to someone else's representation of four and two and asks: "What's the same and what's different?" The learners notice that whilst there are still six children altogether, they have used counters instead of people, they have used colour to show who is at the slide and who is at the climbing frame, they have four people playing on the slide and two people playing on the climbing frame etc.

The teacher asks how to write the different combinations, and they agree together that they can write 6 = 4 + 2, 4 + 2 = 6 and 2 + 4 = 6 (for the example above). The teacher then asks the learners to explain the symbols using their pictures or their resources and they explain and show that if there are four children at the climbing frame and two children at the slide there are six children playing altogether.

As we worked with the connective model in this way, in different contexts and particularly when collaboratively planning with teachers, we started to adjust the words in the model and, over the next 20+ years, the model we used evolved. The labels of 'language' and 'symbols' remain unchanged but 'concrete experiences' and 'pictures' have been adapted and undergone several subtle but deliberate changes.

The connective model 2004: small adjustments

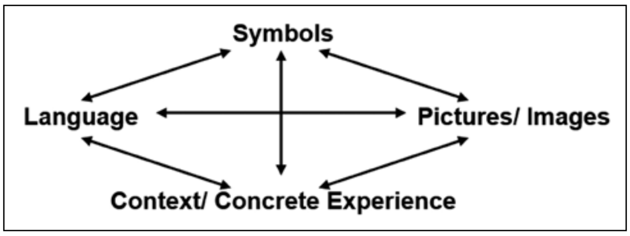


Figure 3: The connective model 2004.

In our early work exploring the connective model with teachers, we remained faithful to Haylock and Cockburn. Over time, we recognised that we were looking at and exploring context as part of concrete experience. We identified that context for the mathematics, especially in relation to sense making, was hugely important and it fitted with concrete experiences, as concrete experiences are often linked to a context. For example, for the context of putting shopping in a basket, linked to understanding the additive structure of the counting system, concrete experiences of the context include using a shopping basket in a real shop, a role play experience in a classroom shop and putting objects in a box.

We also recognised that for some aspects of mathematics it is hard to provide a concrete experience in the classroom that can be viewed and manipulated easily, for example for very large and very small numbers. We have used collections of bottle tops to start building an understanding of the size of large numbers; the experience of creating bags of tops to build a large number provides some understanding of scale but it is slightly contrived and limited. Linking this experience to a context, such as the number of people at the Taylor Swift concert in Edinburgh, the crowd at Wembley to see the Lionesses etc for which we can provide either a virtual experience or draw on a shared experiences from outside of the classroom, bridges the gap left by the limitations of the concrete representation and brings new meaning and understanding to numbers like 73 000. The connecting of the different representations is crucial for developing understanding.

We recognised context was an implicit part of the original model, but we wanted to make it explicit, adding it alongside concrete experience, to acknowledge the importance we thought it had and to avoid it being forgotten or ignored. We were also aware of the limitation of the label 'pictures' and wanted to indicate that the intention is that it is more than photographs or drawings of concrete objects. By including 'images' we felt it more clearly indicated a broader group of two-dimensional representations.

The connective model 2009: a major shift

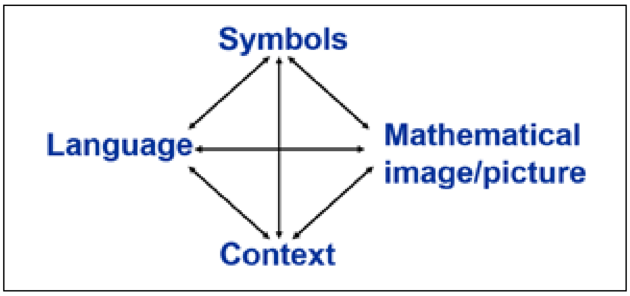


Figure 4: The connective model 2009.

Following its introduction to the model, we continued to explore context with teachers. We noticed context has a dual role: it can help learners access and make sense of the mathematics, and it can also challenge and deepen understanding. It is quite common for us to observe children **and** adults manipulating symbols successfully, with limited understanding of the mathematics behind the symbols, and we have found introducing a context demands they make sense of the symbols. Alongside this, the context also provides a purpose for the mathematics.

An example of this, taken from our work leading professional development programmes, is making sense of a fraction as the result of a division. Starting from the symbols, we ask teachers to draw a picture to show a fraction, such as $\frac{2}{3}$, and then another picture and another. We have observed the following:

1. The first drawings are usually a single shape split into three equal parts with two parts shaded: mainly circles and rectangles, occasionally triangles (Figure 5).

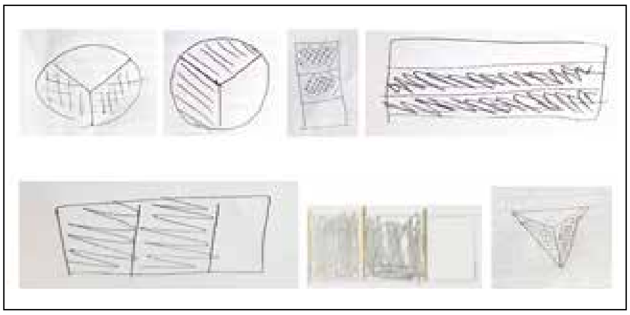


Figure 5: Representations of two thirds.

Sometimes the rectangles are drawn and described as bars (Figure 6):

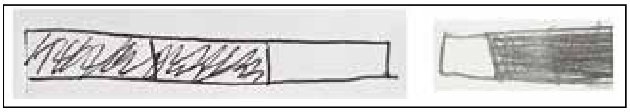


Figure 6: Bars representing two-thirds.

2. When asked to draw, ‘another picture and another’, a fraction of a set is commonly the second or third drawing, usually a set of three objects with something the same about two of them (for example, shading). Sometimes the set has more than three, but most people opt for the simplest case (Figure 7):

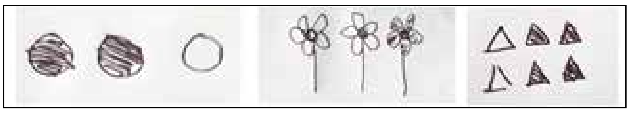


Figure 7: More representations of two-thirds.

We have found it is extremely unusual, in this context, for $\frac{2}{3}$ to be represented as a number on a number line (Figure 8):

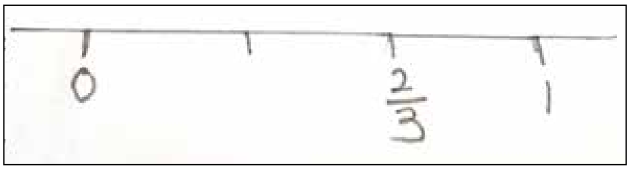


Figure 8: Two-thirds on a number line.

Having asked teachers to draw representations of $\frac{2}{3}$, we then ask them to consider $2 \div 3 = \frac{2}{3}$ and they recognise that none of their drawings match this division. The next step is to represent the calculation and put it in a context.

Many people choose to stick with discrete objects, often in the context of food (cakes, pizzas and bars of chocolate are the favourites) and what then follows is often doubt and confusion. To illustrate this let’s take a commonly used context: two bars of chocolate shared between three people. Typically, these are represented with two rectangles and parts are shaded as in Figure 9:

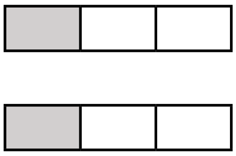


Figure 9: Representation of $2 \div 3 = \frac{2}{3}$.

Asked to explain the two thirds, the exchange below (where A = Adviser and T = Teacher) typifies our experience:

T: You have two bars of chocolate and split each bar into three equal parts.

A: What are the shaded parts showing us?

T: That’s your share, you get a third of each bar of chocolate so that’s two thirds

A: Two thirds of what?

T: Two thirds **of the two bars of chocolate.**

A: Is it two thirds of the two bars of chocolate?

T: Yes…no…no its two sixths of the two bars of chocolate because there are six pieces in total. That’s one third. You get one third.

A: So, the calculation is wrong, two divided by three doesn’t equal two thirds, it equals one third?

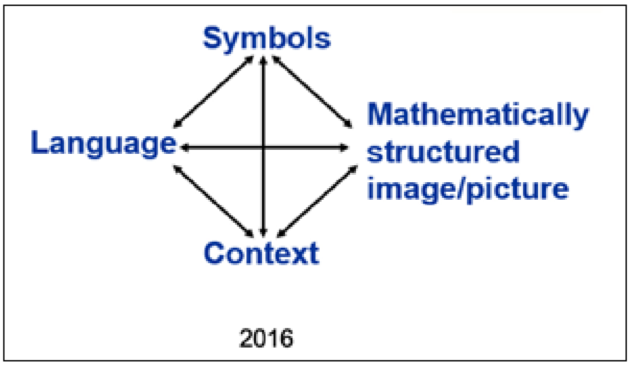
T: I don’t know, I’m confused!

All of these teachers are comfortable with writing $2 \div 3 = \frac{2}{3}$ but putting the calculation into a context, drawing a representation of this context, and then explaining the meaning of the symbols in the context, using the drawing, exposes uncertainty and confusion between finding one third of two and the resulting two thirds of one. Without the requirement to represent with a context, manipulation of symbols can result in correct calculations without understanding.

To support thinking about the meaning of $\frac{2}{3}$ as the result of a division, we choose a continuous context, such as length or capacity, rather than discrete objects. For example, a two-litre bottle of water being shared between three people. Each person will get one third of the bottle, which will be two thirds of one litre of water. This seems to be easier to make sense of; adding the unit to the calculation makes this clear: $2L \div 3 = \frac{2}{3}L$.

This exploration of $2 \div 3 = \frac{2}{3}$ demonstrates how including different representations of the mathematics can lead to deeper understanding when those representations are connected. It shows movement around the connective model, connecting symbols, mathematical image, context and language, as well as connecting different representations of the same type, for example making sense of how different drawings of $\frac{2}{3}$ are connected. Connections within the model are both between the different types of representation and within each type of representation.

The Connective Model 2016 and 2018: fine tuning



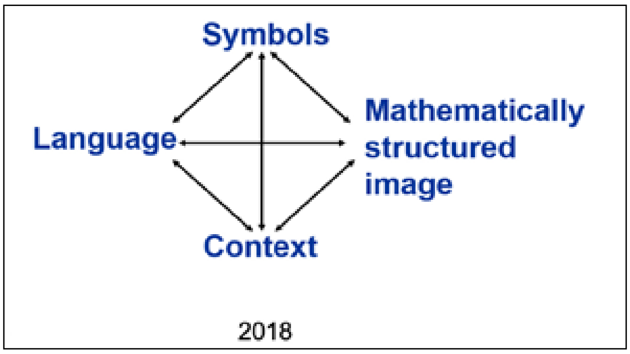


Figure 10: Connective models from 2016 and 2018.

As a consequence of replacing 'concrete' with 'context' we realised that, whilst concrete and pictorial provide one way of sorting and classifying representations, we were sorting and classifying in a different way. Our attention had shifted to the purpose of the representation. Context situates the mathematics and mathematically structured representations draw attention to the structure of the mathematics.

Concrete and pictorial representations have not disappeared from our version of the model, they sit within both context and mathematically structured image. One of our observations from working with the original labels is that learners can have concrete and pictorial experiences without mathematical structure ever being represented. For example, I count some counters and then I draw circles to show what I have counted. No structure, nothing to notice.

However, if we consider shifting the focus to context and mathematically structured images, now I collect some stones for the game *Six Nice Things* and I want to know that I have six. I place the stones in the holes of a Numicon plate, or I arrange the stones on a tens frame, so that I can see there are six and so can other people (Figure 11).

Figure 11: Six nice things.

Exposing and attending to mathematical structure is necessary for developing deep understanding and our relabelling prioritises this idea. Concrete experiences in context often lack mathematical structure but you can choose to impose a structure on them, for example arranging the stones on a tens frame, linking concrete and mathematically structured images.

Language and symbols

Whilst we have left the labels 'language' and 'symbols' unchanged, our understanding of them has developed over the years.

Initially we thought language indicated mathematical words, the technical vocabulary, but we now understand it as much more than this. We have found that forming sentences using mathematical language is where the challenge sits, understanding the grammar surrounding how technical words are used to communicate mathematical meaning.

A focus on lists of words in maths can be problematic for several reasons:

- They can suggest to learners that maths is about finding the right label and lead to them offering individual words as answers, in the hope that they come up with the right one, rather than explaining their thinking in sentences.
- Words have multiple meanings; for example, if

the word 'more' is on a list beside the symbol +, the use of 'more' in the question '*If I have £73 and you have £68, how much more than you do I have?*' may direct learners to solve the problem by adding 73 and 68.

- A focus on context brings with it contextual language that needs to be understood and connected to other representations of the mathematics. For example, additive contexts might give rise to the language of 'getting on' and 'getting off' (a bus), 'getting in' and 'getting out' (of a swimming pool) and 'going up' and 'going down' (steps), language that will not be found on lists of mathematical vocabulary but connects the mathematics with everyday experiences.
- Lists can imply that each symbol represents one word on the list and symbols should be 'read' left to right.
    - o Separately labelling each symbol can obscure the meaning of expressions and equations and so limit ways of thinking about the mathematics expressed by the symbols. For example: 603 – 598; understanding may be restricted to 'six hundred and three (603) take away (-) five hundred and ninety-eight (598)', leading to using a method to 'take away' 598. Where language is used to make sense of the symbols, the different ways you can think about them, then the same calculation can be thought about as 'How close together are six hundred and three and five hundred and ninety-eight?' leading to a very simple mental calculation.
    - o Separately labelling each symbol can also create a barrier to understanding and symbolically representing problems that are not presented in the order the symbols would be written; for example, *I am three years younger than my sister, who is twelve. How old am I?* is represented as 3 – 12 = to keep the numbers in the same order as the problem.

This last problem is one of the reasons we have noticed that the challenge for many learners is they experience symbols as unconnected with their other mathematical experiences. Separately memorising how to manipulate the symbols can become the focus, rather than the symbols being embedded with meaning arising from making sense of and communicating other experiences and representations. Without meaning, relying on memorising makes a huge demand on executive function skills and steers learners away from mathematical thinking.

The connective model has defined the thinking of our team for nearly a quarter of a century, and we are grateful to Derek Haylock and Ann Cockburn for starting us on this pedagogical journey. It is hard for us to imagine life without the connective model, it is in our mathematical blood and flows through our work. And we believe it has much to offer teachers for at least the next 25 years.

References

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