

# **The impact of the teaching of multiplication and the multiplication tables check**

## **Final report June 2025**

Report of a research study in England and Jersey

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## **The impact of the teaching of multiplication and the multiplication tables check**

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### **Introduction**

The Multiplication Tables Check (MTC) for year 4 pupils in England was announced by the Department for Education (DfE) in September 2017. This new assessment was intended to determine whether pupils can fluently recall their multiplication tables up to 12 x 12. The MTC is statutory for all year 4 pupils registered at state-funded maintained schools, special schools, or academies, including free schools, in England. Voluntary trialling of the MTC was offered to schools in June 2019 ahead of the test becoming statutory from June 2020.

The MTC is an online assessment which schools are required to administer to pupils either on a computer or a tablet during a 3-week assessment window in June. The check consists of 25 questions with 6 seconds given for each question. Not all pupils need to take the check on the same day. Questions are offered as multiplications using only symbols in the form 7 x 8; division and other representations of multiplicative relationships are not included.

A total score out of 25 is reported to each school for all pupils who take the check. There is no expected standard; the number and percentage of pupils who achieve full marks is reported. Individual pupil and school-level results are made available to schools and these results are also made available to Ofsted, via the Analyse School Performance (ASP) data system. The results are not published in performance tables. In January 2024 Ofsted updated the School Inspection Handbook to reference the MTC. Paragraph 273 was included to allow inspectors to gather information about the extent to which pupils recall their times tables fluently.

### **Rationale for the research**

During a workshop at the joint conference of the Association of Teachers of Mathematics (ATM) and Mathematical Association (MA) 2022, a conversation occurred between three participants around the validity and necessity of the MTC for pupils in England. This was prompted because pupils in Jersey are not required to take the MTC. Two of the three participants were mathematics advisers from England and had evidence and experience of the MTC while the third participant was a mathematics adviser in Jersey, reflecting on the decision for their pupils (who participate in the National Curriculum KS2 tests) NOT to take the MTC. While the conversation was animated and at times persuasive, in favour of not taking the MTC, there was no clear evidence or data to either support or refute the claims that using the MTC would improve pupils' fluency and mathematical understanding.

This was the catalyst to form a research project between England and Jersey to investigate pupils' understanding of multiplicative relationships. In particular, we were keen to understand if the inclusion of the MTC had a positive impact on pupils who take it, in England, in terms of their understanding of multiplicative relationships, compared to pupils in Jersey who do not have the MTC.

The research team was initially six education/mathematics advisers, one based in Jersey, one based in Norfolk and four based in Devon. They were joined by research assistants supported by Professor Emily Farran at the University of Surrey.

## Fluency

There is a difference between recognising number relationships you know and understanding how and when to use number relationships you know. Fluency requires both.

The current English National Curriculum (DfE, 2013) echoes this, with fluency as its first of three aims:

*... become **fluent** in the fundamentals of mathematics, including through varied and frequent practice with increasingly complex problems over time, so that pupils develop conceptual understanding and the ability to recall and apply knowledge rapidly and accurately.* (p. 2)

In the Jersey Mathematics Curriculum (Government of Jersey, 2018) the wording is almost identical except the word rapidly is replaced by effectively:

*...become fluent in the fundamentals of mathematics, including through varied and frequent practice with increasingly complex problems over time, so that pupils develop conceptual understanding and the ability to recall and apply knowledge effectively and accurately.* (p. 134)

However, over the years we have experienced fluency becoming more synonymous with words such as practice, fast, rote, and automatic and less so with words such as flexible, making connections and understanding.

When thinking about the design of the project, we were influenced by several key pieces of research. The first was a study by Anghileri et al. (2002) which compared English and Dutch upper primary school pupils and their strategies for completing division questions. Their sample involved 534 pupils aged nine and ten years. Overall, the Dutch pupils were more successful in gaining correct responses to questions and showed more progression over time. While the English pupils were often disorganised and it was difficult to follow their recording, the Dutch pupils were more organised and displayed a clearer use of informal methods. Support for structuring informal strategies appeared to be more effective than replacing them with formal methods, compared to the English pupils who showed a discontinuity between informal strategies and traditional algorithms.

Another study that we drew on was a ten-year long study by Borthwick and Harcourt-Heath (2006, 2008, 2010, 2012, 2016) who explored calculation strategies Year 5 pupils chose to use to solve abstract addition, subtraction, multiplication, and division questions. Repeatedly, their research showed that when pupils selected a strategy based on a mental method, they usually reached a correct response, compared to pupils who used a formal, traditional method.

Finally, a working party within the Primary ATM and MA Group produced a position statement (2021) on the teaching and learning of multiplication bonds (also referred to as times tables), prompted by the introduction of the MTC for Year 4 pupils in England. This captured views of members of the group around the debate of whether pupils need to have automatic recall with multiplication bonds. They identified '*flexibility and decision making*' as key elements of fluency and suggested that when pupils have automaticity, as opposed to memorisation, they are enabled to '*understand multiplicative relationships, make connections and build fluency*'.

## Research questions

Our main research question is:

- Do pupils in England, who have the multiplication tables check, demonstrate more understanding and knowledge of multiplication than pupils in Jersey, who do not have the multiplication tables check?

Further questions which emerged from the data included:

- Is the emphasis on learning multiplication facts undermining understanding the multiplicative relationship?
- Is the emphasis on learning multiplication facts leading to a lack of understanding of contexts for multiplication?
- Is there an emphasis on written methods for multiplications and divisions outside of the multiplication tables?

## Data collection

We were keen to ensure equity between schools and pupils across both England and Jersey. There are 24 primary schools in Jersey, and we invited the whole sample to participate. For parity we agreed to try and recruit the same number of schools in England. Researchers from England were from two different geographical locations, and so we decided to recruit 12 schools in each. We analysed the demographics of the 24 Jersey schools (e.g. Pupil Premium, SEND, number on roll) and used this data to identify schools in England. We resulted in a total of 37 participating schools: 17 from Jersey and 20 from England.

Schools and teachers in England were recruited predominantly through previous contact with each researcher, and we do recognise that this presents a conscious bias within the choice of these schools and their data. All data collected was anonymous and sent to one data controller. Schools, teachers, and pupils had the right to opt out of the study at any time prior to the anonymous data being shared.

Originally two year groups were selected: Year 4 and Year 5. Our reasons were that Year 4 is when the MTC is administered and so was an obvious choice, but we were also keen to see if the results had any impact for the following year too. Once we had collected the data, we realised that the sample was too large for this scale of project. Data was collected in the Autumn term, which meant that pupils in Year 4 had only experienced this year group for around 10 weeks, and so we decided to focus on data from Year 5 pupils only. These pupils had received a year of teaching that led to taking the MTC.

## Summary of data

### Total number of Schools

Number of schools in Jersey: 17

Number of schools in England: 20 (10 from Devon and Cambridge, 10 from Norfolk/Suffolk)

Total number of schools in the project: **37 schools**

## **Total number of Pupils**

Number of pupils in Jersey: 1334

Number of pupils in England: 1879 (1061 in Devon, 818 in Norfolk/Suffolk)

Total number of pupils in the project: **3213 pupils**

**Data received (some data did not arrive) 2039**

## **Total number of pupils in Year 4**

Number of Year 4 pupils in Jersey: 539 pupils

Number of Year 4 pupils in England: 455 pupils

Total number of pupils in Year 4: 994 pupils

## **Total number of pupils in Year 5**

Number of Year 5 pupils in Jersey: 511 pupils

Number of Year 5 pupils in England: 534 pupils

Total number of pupils in Year 5: 1045 pupils

## **Design of the study**

### **Maths papers**

#### **1. Choice of questions**

The research team made the following decisions about the questions in the mathematics papers for both Year 4 and Year 5:

- ***Limiting the papers to ten questions***

This was agreed early on; it reflected the number of questions in the Anghileri et al (2002) study and allowed for variation within the questions whilst still being short enough papers to avoid being an imposition on teachers and learners.

- ***Inclusion of both multiplication and division questions***

The purpose of reaching automaticity with multiplication bonds is so that these bonds can be used to solve multiplicative problems. This includes problems involving multiplication **and** problems involving division. The decision to have a mixture of questions involving multiplication and division was therefore deliberate, including a pair of linked questions in each paper (see *inclusion of linked questions* below).

- ***Inclusion of questions with different representations of the mathematics***

Interpreting different contexts as multiplicative is key to using multiplication bonds to solve problems and a deep understanding of multiplicative relationships includes representing them in different ways. This led to the design of the questions as follows:

- Question 1: starting from a picture and matching symbolic representation of both multiplication and division to the picture.



- Questions 5, 6, 7, 8 and 9: starting from symbols with a symbolic response required. The reason for five questions using symbols is explored below.
- Questions 2, 3 and 4: starting from contexts, given in words, with a symbolic response required. Question 6 also includes a picture to provide further understanding of the context.
- Question 10: starting from symbols, with a context, given in words, the required response. This question was the same on both papers as the challenge was in constructing a context it was felt unnecessary to vary the numbers and provided an opportunity for comparison across the year groups.

- ***Inclusion of questions outside of the multiplication tables***

As stated above, the purpose of reaching automaticity with multiplication bonds is so that these bonds can be used to solve multiplicative problems. This includes:

- Questions 8 and 9: problems where the numbers sit outside of the multiplication tables and solutions are derived by applying understanding of both multiplication bonds and multiplicative relationships. This is reflected in the multiplication in question 8 and the division in question 9.
- Y5 question 4: a division where the dividend is **NOT** a multiple of the divisor.

- ***Inclusion of linked questions***

Automaticity with multiplication bonds includes making links between multiplications and divisions, including when they are presented in different ways; understanding the multiplicative relationship between a trio of numbers is a crucial part of fluency. This is reflected in decisions made for:

- Question 1: the same picture is expected to be connected to both a multiplication and a division.
- Questions 1 and 7: the pictorial representation of a multiplicative relationship in question 1 involves the same numbers presented as a division in question 7.
- Questions 3 and 6: the relevant calculation needed to solve the problem for question 3 matches the multiplication in question 6.
- Year 4 questions 5 and 9: the multiplication in question 5 can be used to derive the answer to the division in question 9.

- ***Inclusion of questions with excess information***

Making sense of mathematics and solving problems draws on executive function skills. Sometimes, the need for these skills is stripped out of questions, resulting in learners ignoring the words and the context and simply finding the numbers and operating on them. This led to the decision to make:

- Question 2 a problem containing only two numbers, that need to be multiplied together to solve the problem.
- Question 3 a problem containing three numbers, with the number that is not needed appearing between the two numbers that need to be multiplied together to solve the problem.

## 2. Structure

- For each year group a recorded ppt was provided so that all participants in the study heard and saw the same questions, read in the same way, and were allocated the same length of time for answering the questions.
  - The recorded ppt started by thanking those watching for their help with the project and provided the following information:  
*I'm going to be asking you some questions that will also appear on the screen, and you have a paper for writing your answers. You can write and draw anything you want to, to help you find an answer. We're looking forward to seeing how each of you chooses to do this.*
  - A practice question,  $50 + 50$ , was used for both year groups and teachers then paused the ppt, to answer any questions.
  - An answer sheet was provided with a space for each question.

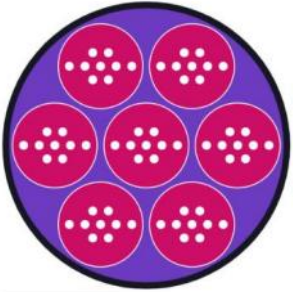
1.	$\underline{\hspace{2cm}} \times \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$ $\underline{\hspace{2cm}} \div \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$	
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Figure 1: Year 5 Question 1 Answer sheet

- Question 1: the answer paper included the image for question 1 and two calculations to complete
- Question 10: the calculation appeared on the paper.

10.	$3 \times 7 = 21$
-----	-------------------

Figure 2: Year 4 and Year 5 Question 10 Answer sheet

- Question numbers appeared on the slides and were read out.
- Each question with words was then read twice, whilst those with just symbols were read once.
- Multiplied by', 'Divided by' and 'Equals' was the language used for the questions involving symbols.

- The reading of each question was followed by a pause before the slide moved on, with approximate timings (shown in the table 1).

Question	Reading out time	Pause	Total time on screen
1	17 seconds	20 seconds	37 seconds
2	16 seconds	10 seconds	26 seconds
3	20 seconds	20 seconds	40 seconds
4	18 seconds Y4 16 seconds Y5	20 seconds	38 seconds Y4 36 seconds Y5
5	5 seconds	6 seconds	11 seconds
6	5 seconds	6 seconds	11 seconds
7	5 seconds	6 seconds	11 seconds
8	6 seconds	10 seconds Y4 11 seconds Y5	16 seconds Y4 17 seconds Y5
9	5 seconds Y4 7 seconds Y5	11 seconds Y4 14 seconds Y5	16 seconds Y4 21 seconds Y5
10	14 seconds	32 seconds	46 seconds

*Table 1: Timings for questions on recorded PPT*

- After five questions there was an instruction to 'please turn over your paper'.
- At the end of the ppt was the message: *Thank you for helping us with our project.* (see Appendices 1 and 2 for the full set of questions for Y4 and Y5)

## Questionnaires

In addition to responses to maths questions, the team was interested in current thinking about multiplication and the impact on both practice and attitudes. Two short questionnaires were devised, one for pupils (the same for both Y4 and Y5) and one for the class teachers (see Appendices 3 and 4).

## Structure

Participating teachers were asked to complete all the tasks in the week beginning 21<sup>st</sup> November 2022 as follows:

- Maths questions first: this was chosen so that exploring attitudes to learning multiplication facts did not influence or impact on the responses to the mathematics questions and it meant the learners were at their 'freshest' for the maths questions.

- Pupil attitude questionnaire: it was suggested that this was done after the maths papers had been collected and a recorded ppt was provided for these questionnaires. There were six questions that required ticking or circling responses on the sheet provided.
- Teacher attitude questionnaire: to be completed at any time during the week.

## Data

The papers were returned to one address. The first attempt at processing the data revealed several significant issues:

- The research team agreed detailed coding for the maths papers, that reflected the thinking behind each question, including the choice of numbers. This meant entering data was time consuming. For example, for question 3 on the Year 5 paper, each response had to be matched to one or more of the following nine possible codes:
  - Blank
  - Wrote 11 x 12 or 12 x 11
  - No answer
  - Other calculation
  - Used 4 in the calculation
  - Correct 132
  - Incorrect
  - Counted
  - Drawing

Each response to question 3 could lead to multiple entries; for example, a participant may have written the calculation 11 x 12 and started to draw sets of dots but then abandoned the question, leaving them with no answer.

- The research team had underestimated the quantity of data that would be produced. With over 2 000 papers received and ten questions on each maths paper, this meant over 20 000 responses to maths questions to process.
- The data was anonymised to the extent that there was no way to match maths papers with questionnaires. This put a limit on the analysis that was possible.

## Sample data

To provide some early analysis that could be shared with peers, it was decided to focus on Year 5 and a sample of around 200 papers from England and 200 papers from Jersey were processed using the detailed coding (see Appendix 5).

The research team examined the sample data and some conjectures emerged. These were used to shape a workshop for the Joint Conference of Mathematics Subject Associations held on April 3<sup>rd</sup> and 4<sup>th</sup> 2023.

The workshop provided:

- a background to the study
- an opportunity for the participants to work on one of the maths papers and then reflect on the experience, sharing thoughts and observations.
- exploration of three emerging questions with examples from the sample papers:
  - Is the emphasis on learning multiplication facts undermining understanding the multiplicative relationship?
  - Is the emphasis on learning multiplication facts leading to a lack of understanding of contexts for multiplication?
  - Is there an emphasis on written methods for multiplications and divisions outside of the multiplication tables?

With the quantity of data collected, the research team had decided that it would only be possible to process a sample and sought advice on how this should be done. However, participants in the workshop were keen for all the data to be processed and suggested sources of help. This led to a meeting in May 2023 with research colleagues; Camilla Gilmore at Loughborough University and Emily Farran at the University of Surrey.

This support provided direction for the analysis of the data:

- Processing of Year 5 data from the maths papers to happen at three levels:
  - All responses to be processed with a simplified coding by a research student supported by a member of the research team. This coding would indicate whether there was a correct answer or not and would allow for a comparison between England and Jersey.
  - A sample of papers to then be used with the detailed coding and processed by a member of the research team, as the coding requires specialist knowledge. The sample to be decided once the full data had been processed and analysed as the analysis might indicate the sample that would be of most interest.
  - Linked to themes emerging from the detailed coding, a case study approach to examine these in more detail.
- Processing of data from attitude questionnaires proved a much simpler process which did not require specialist knowledge; a member of the DES admin team was able to provide the time to complete this task.

### Data analysis: pupils' responses

Question	1a	1b	2	3	4	5	6	7	8	9	10
England	56	45.3	68.4	46.4	12.92	58.4	70.4	66.7	32.4	15.2	18.7
Jersey	53.4	45.2	66.7	55.2	18.2	61.4	69.1	63.8	35.6	11.4	13.3

Table 2: Percentage of Year 5 pupils with correct responses for questions 1 to 10, by country

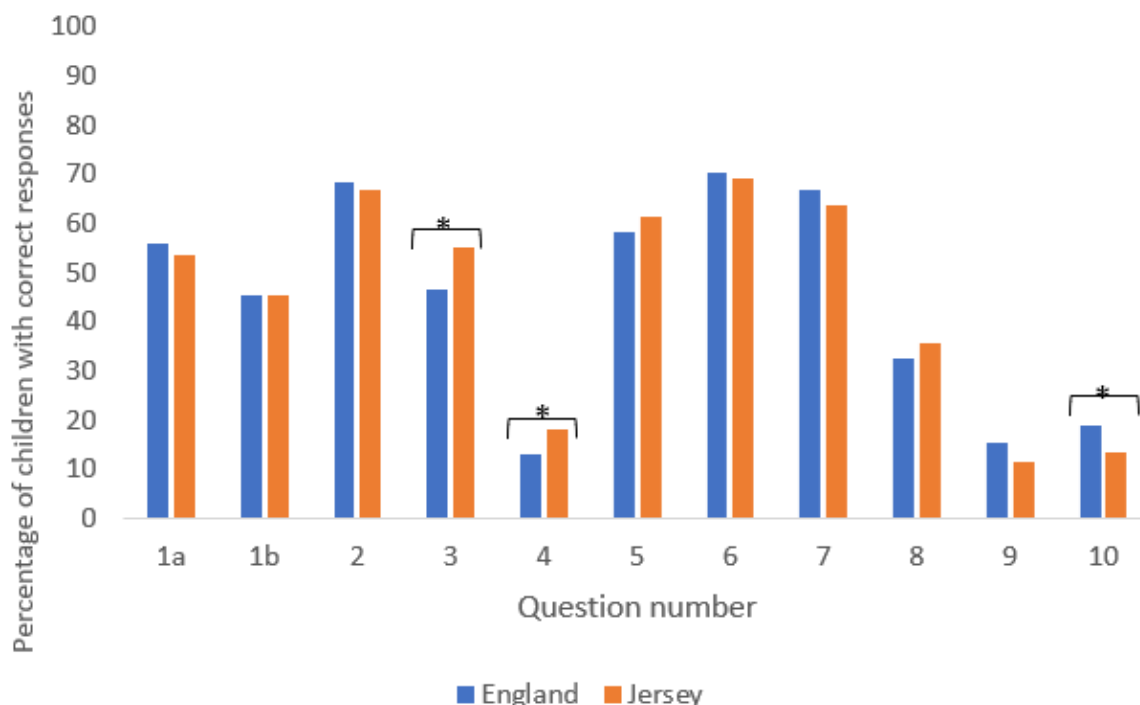


Figure 3: Percentage of Year 5 pupils with correct responses for questions 1 to 10, by country

Question number	Chi squared	df	p-value
1a	0.695	1	0.4045
1b	0.00134	1	0.9708
2	0.313	1	0.5760
3	8.09	1	0.0045
4	5.55	1	0.0184
5	0.992	1	0.3192
6	0.22	1	0.6394
7	0.949	1	0.3300
8	1.21	1	0.2720
9	3.30	1	0.0692
10	5.75	1	0.0165

Table 3: Chi-squared statistics for questions 1 to 10

## Section 1

The results from chi-squared analyses revealed that out of the eleven questions, only three showed that the proportion of pupils providing the correct responses significantly differed by country (Table 3 and indicated by  $\chi^2$  on Figure 3).

A higher proportion of Jersey Year 5 students provided the correct response than the England Year 5 students for questions 3 ( $p=.0045$ ) and 4 ( $p=.0184$ ). These questions are both contextual questions, one linked to a multiplication and the other linked to a division, both within the multiplication tables. This provides some support to the hypothesis that an emphasis on learning multiplication facts in England is resulting in a lack of understanding of contexts for multiplication, compared to the pupils in Jersey. The level of correct responses to the division in Q4 was very low in both countries, suggesting that teaching in both jurisdictions may be focused on multiplication bonds and multiplication divorced from understanding the multiplicative relationship between numbers and how multiplication and division are both part of that relationship. Q4 also involved making sense of a remainder in a context.

A higher proportion of England pupils provided the correct response than the Jersey pupils when answering question 10 ( $p=.0165$ ). This question asked pupils to provide a written problem for the equation  $3 \times 7 = 21$ . However, correct response levels were low in both areas; that all pupils found this question difficult.

## Distribution of scores

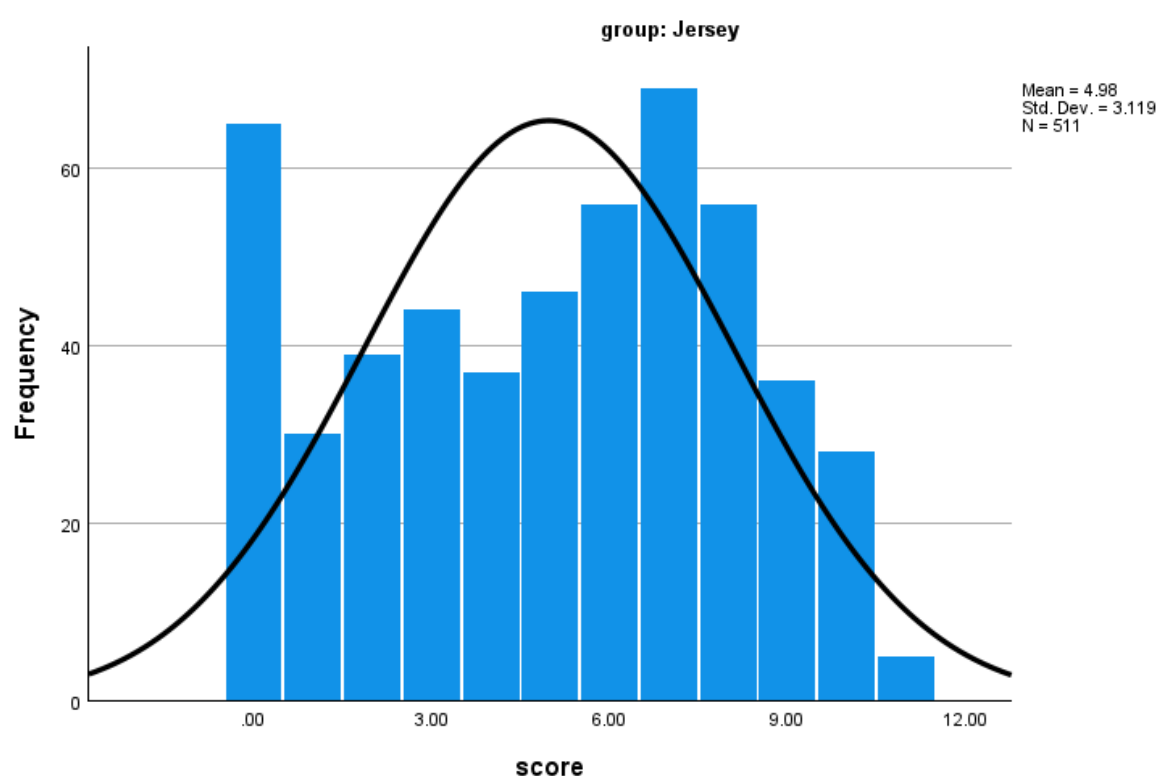
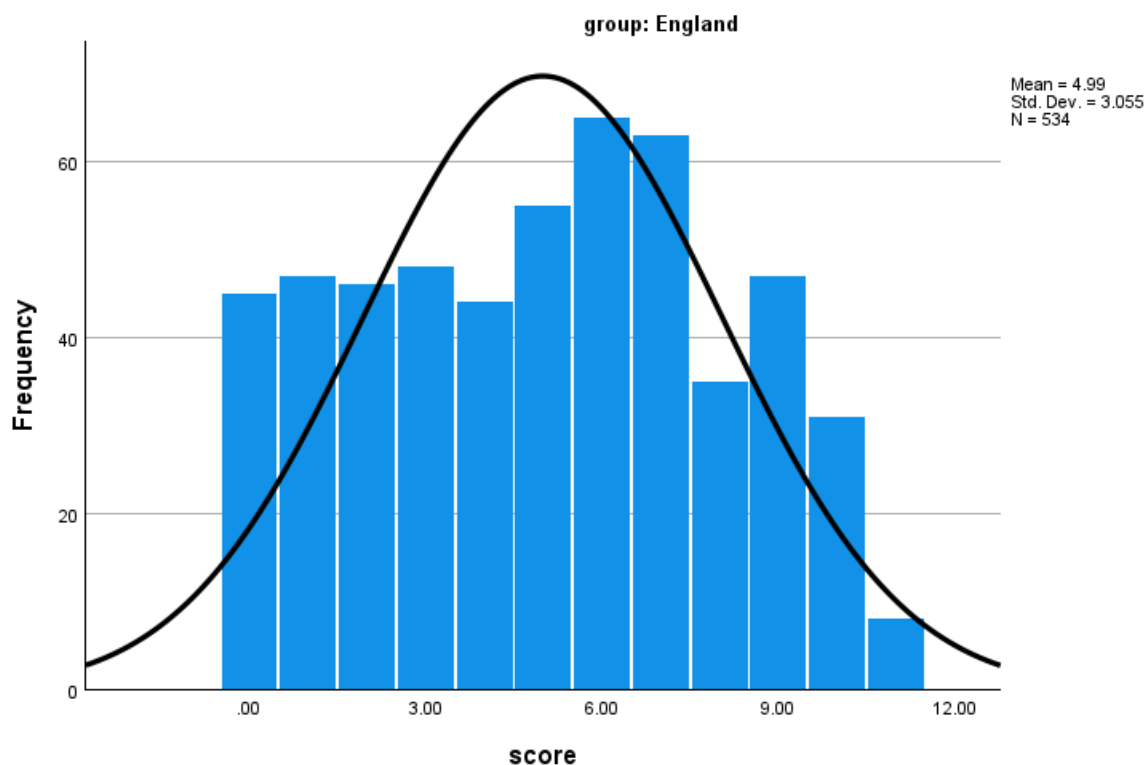


Figure 4: Distribution of scores in Jersey



*Figure 5: Distribution of scores in England*

Distribution of scores showed very little difference between the two jurisdictions as shown in figures 4 and 5; the main indication is that there was very little difference in performance between England and Jersey which suggests no positive impact of the MTC. More concerning, there is an indication that in both jurisdictions, the teaching of multiplicative reasoning is not leading to an understanding of multiplicative relationships in different contexts.

## **Section 2: Is the emphasis on learning multiplication facts leading to a lack of understanding of contexts for multiplication?**

This emerging research question can be addressed in three ways. First, question 3 and question 6 were designed for direct comparison, a symbolic and a contextual example of the same multiplication.

### **Section 2a: Proportion of correct responses for questions 3 and 6, by country**

A three-way loglinear analysis of country (England, Jersey) by question (question 3, question 6) by response (correct, incorrect) produced a final model that retained the two-way interactions only. The likelihood ratio of this model was  $\chi^2(1) = 2.068$ ,  $p = .150$ . For this analysis, a non-significant p-value indicates a good fit of the data to the model. The largest standardized parameter estimate in the model was for the interaction between questions 3 and 6,  $Z = 14.996$ ,  $p < .001$ . This significant interaction indicates that being correct on one of these two questions is associated with being correct on the other question, and vice versa for incorrect answers. This 2-way effect did not interact with country ( $Z = -1.459$ ,  $p = .145$ ), indicating that pupils from England are just as likely as pupils from Jersey to provide similar answers to question 3 and 6. This does not support the hypothesis that the MTC has resulted in more of an emphasis on multiplicative facts in England leading to a lack of understanding of contexts for multiplication, but the results do seem to show that this is an issue in both countries. The remaining two-way interactions are discussed in section 1 above; chi-squared analyses of question 3 by country

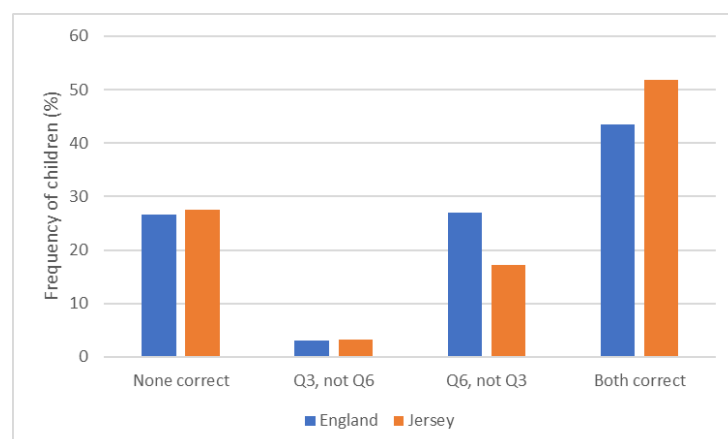


and question 6 by country indicated that correct responses were more likely for question 3 (the contextual question) in Jersey than in England, but that there were no differences between Jersey and England for question 6 (the equivalent symbolic question). Whilst this offers some support for the hypothesis, it must be considered within the context of the non-significant 3-way interaction (Table 4).

Country				Q6		
				Incorrect/no answer	Correct	Total
England	Q3	Incorrect/no answer	% of total	26.6	25.8	52.4
		Correct	% of total	3.0	<b>44.6</b>	47.6
Total			% of total	29.6	70.4	100
Jersey	Q3	Incorrect/no answer	% of total	27.6	16.0	43.6
		Correct	% of total	3.3	<b>53.0</b>	56.4
Total			% of total	30.9	69.1	100

*Table 4: Contingency table of country by question by response*

Whilst the table reveals over 40% of pupils in both jurisdictions got both Question 3 and Question 6 correct (England 45%, Jersey 53%) and over a quarter of the pupils in both jurisdictions got neither question correct (England 27%, Jersey 28%) we carried out further analysis on the responses from pupils who got only one of the questions correct.



*Figure 6: Responses to Question 3 and Question 6, involving the same multiplication*


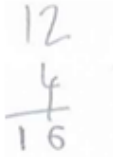

Q3 responses	Jersey	England	Examples
Multiplication using 4: $11 \times 4$ $12 \times 4$	22/82 27%	31/142 22%	
Used addition $11 + 4$ $12 + 4$ $11 + 12$ $11 + 12 + 4$	0/82 0%	11/142 8%	
Blank/ <del>2</del> /don't know/crossed all workings out	26/82 32%	39/142 27%	
121 $11 \times 11$	7/82 9%	6/142 4%	
Others	27/82 33%	55/142 39%	<p>over + 1</p> <p><math>12 \times 11 = 132</math></p> <p><math>12 \div 4 = 3</math></p> <p><math>12 \times 11 = 132</math></p> <p><math>12 \times 12 = 144 - 1 = 143</math></p> <p><u><math>12 \times 11 = 2300</math></u></p> <p>32</p> <p><math>3 \div 2 =</math></p> <p><math>\begin{array}{r} 0 \times 2 \\ 4 \\ \hline 8 \end{array}</math> 8 children in each team.</p> <p><math>\begin{array}{r l} 1 &amp; 10 \ 2 \\ \hline 1 &amp; 10 \ 2 \end{array}</math> 12 21</p> <p>x 1 remainder 1</p> <p>HARD!</p>

Table 5: Incorrect responses to Question 3 by pupils who answered Question 6 correctly

Very few pupils got Question 3 correct but not Question 6 (Figure 6), but a large number got Question 6 correct and Question 3 incorrect with a significant difference between the two jurisdictions: Jersey (16%), England (26%).

Analysis of the responses to Question 3, from pupils who had Question 6 correct and Question 3 incorrect, shows a wide variety (Table 5) with few recognising the calculation as being the one symbolically represented in Question 6, and even where they do identify the same calculation, a connection is not made to the multiplication bond they use in Question 6. This prompts questions about whether pupils understand when the multiplication bonds they have learnt, to automaticity, might be used and whether some are looking to use these only in response to multiplications as presented in the MTC (e.g.  $12 \times 11$ ).

## Section 2b: group differences in symbolic and contextual scores.

For the second and third analyses, questions 2, 3 and 4 were categorised as contextual while questions 5, 6 and 7 were categorised as symbolic. This enabled a score out of 3 to be derived for contextual responses and a score out of 3 for symbolic questions. These were used to determine group differences in scores, and associations between scores, per group.

Data were not normally distributed (Kolmogorov-Smirnov,  $p < .05$ ). However, ANOVA is robust to violations to assumptions of normality and due to the large sample size parametric analyses were deemed appropriate here (Central Limit Theorem, Field, 2013). Using scores (out of 3) for symbolic and contextual question accuracy, ANOVA of country (Jersey or England) by question type (contextual or symbolic) was carried out with score as the dependent variable. There was no significant main effect of country,  $F(1,1043) = .920$ ,  $p = .338$ ,  $\eta_p^2 = .001$ . The main effect of question type was significant,  $F(1,1043) = 441.914$ ,  $p < .001$ ,  $\eta_p^2 = .298$  due to higher contextual scores than symbolic scores. There was a significant question type by country interaction,  $F(1,1043) = 5.594$ ,  $p = .018$ ,  $\eta_p^2 = .005$ .

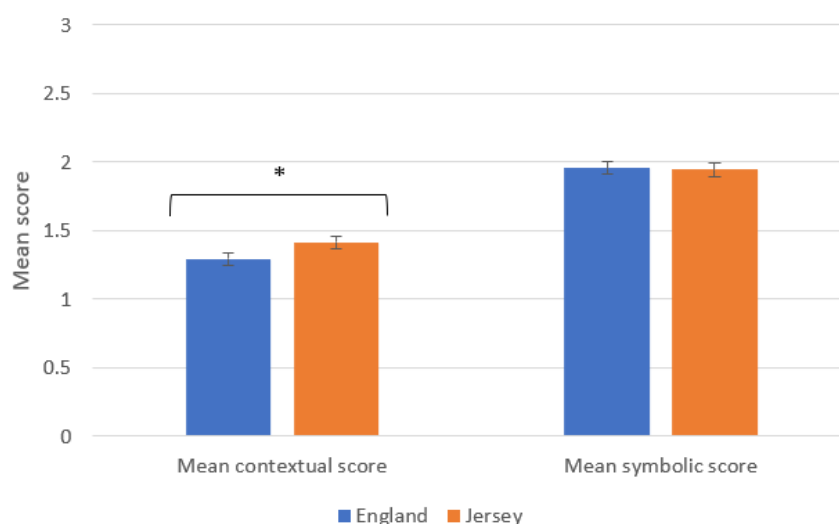


Figure 7: Mean contextual and symbolic scores, by country

To understand the interaction, two independent samples t-tests were carried out to compare the responses for pupils in England vs. Jersey, for each question type. These showed that the source of the interaction was an effect of group for contextual questions, but not symbolic questions. That is, more pupils from Jersey than England answered the contextual questions correctly,  $t(1043) = -2.024$ ,  $p = .022$ . This was not significant for the symbolic questions,  $t(1043) = .174$ ,  $p = .431$  (Figure 7). This supports the hypothesis that an emphasis on learning

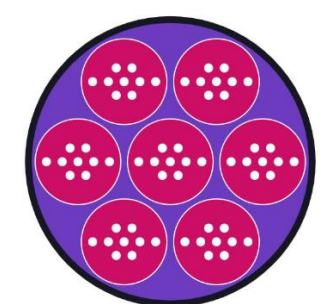
multiplication facts in England, in preparation for the MTC, may be contributing to a lack of understanding of contexts for multiplication as pupils from England perform worse on contextual questions.

### Section 2c: associations between symbolic and contextual scores, by country.

The above group difference approaches assume that the groups are well matched on extraneous variables which might impact level of ability. Although the schools were well matched on available parameters, a prudent approach is to also investigate within country correlations. To this end, non-parametric correlations were performed on the same data as in section 2b to determine whether there is a relationship between contextual and symbolic scores, i.e., does answering the contextual questions correctly mean you are more likely to answer the symbolic ones correctly as well. This showed a significant positive correlation between contextual and symbolic questions for both England ( $r=.581, p<.001$ ) and Jersey ( $r=.637, p<.001$ ). Thus, regardless of country, there is a relationship between the score pupils receive on the contextual questions and on the symbolic questions. This somewhat mirrors the association between question 3 and 6 in section 2a. Any group differences in section 2b should be considered within the context of the significant correlations for both countries outlined here.

### Section 3: Is the emphasis on learning multiplication facts undermining understanding the multiplicative relationship?

Question 1 (Figure 8) is split into two parts: a multiplication (1a) and a division (1b). A three-way loglinear analysis of country (England and Jersey) by question (1a or 1b) by response (correct, incorrect) produced a final model that retained one two-way interaction only, the interaction between question 1a and 1b. The likelihood ratio of this model was  $\chi^2(4)=3.623, p=.459$ , which shows that the model fits the data. The standardized parameter estimate for the significant interaction between question 1a and 1b was significant,  $Z=13.662, p<.001$ . This interaction indicates that being correct on one of these questions is associated with being correct on the other. This 2-way effect did not interact with country ( $Z=-.772, p=.440$ ), demonstrating that pupils from England are just as likely as those from Jersey to provide correct answers to both parts of question 1 (Table 5).



Write down a multiplication fact and a division fact represented by this diagram, which is on your sheet.

Figure 8: Question 1, Year 5

Again, whilst there is little difference in performance between England and Jersey, in terms of getting both parts of question 1 correct, there is an indication that, in both jurisdictions, the teaching of multiplicative reasoning is not securing an understanding of multiplicative relationships. 11% of pupils in England and 8% of pupils in Jersey wrote a correct multiplication without writing a matching division; a secure understanding of multiplicative relationships would result in a division being matched to a multiplication for the diagram.

Country				Q1b		
				Incorrect/no answer	Correct	Total
England	Q1a	Incorrect/no answer	% of total	43.4	0.6	44.0
		Correct	% of total	<b>11.2</b>	44.8	56.0
Total			% of total	54.7	45.3	100
Jersey	Q1a	Incorrect/no answer	% of total	46.4	0.2	46.6
		Correct	% of total	<b>8.4</b>	45.0	53.4
Total			% of total	54.8	45.2	100

Table 6: Contingency table of country by question by response

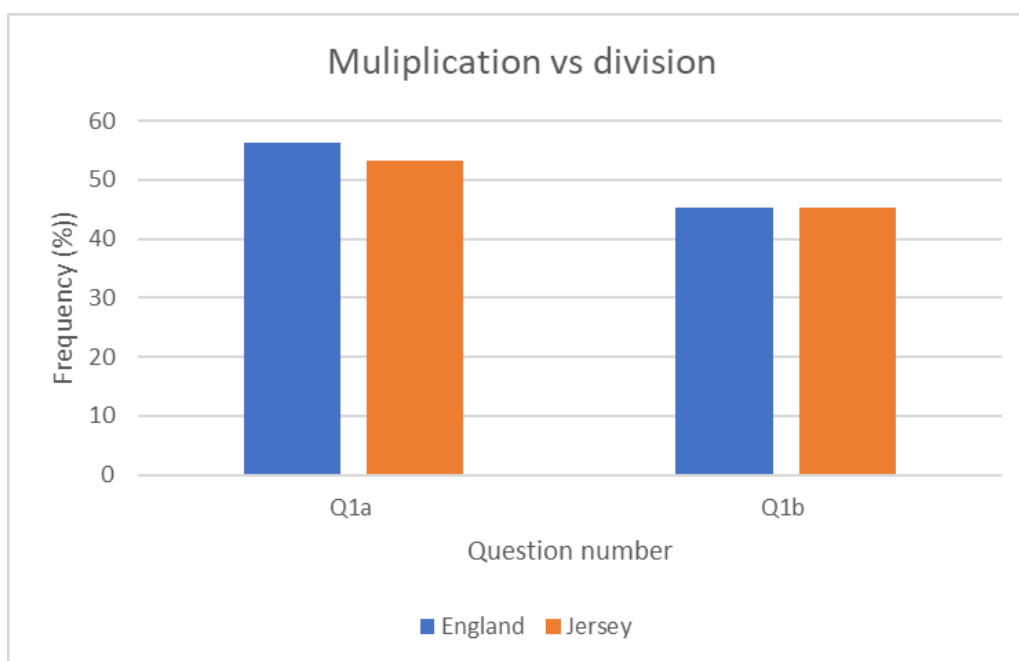


Figure 9: Question 1a (multiplication) and 1b (division), Year 5

Further analysis of the responses from pupils who gave a correct multiplication but did not give a matching division was undertaken (Table 7). The most common response to the division, in these cases, was to leave it blank, again suggesting these pupils were not making connections between multiplication and division, and most responses did not use the same three numbers as the multiplication.

Q1b responses	England	Jersey
Blank or crossed out	25 (40%)	11 (28%)
Three numbers 9/7/63 used incorrectly: $7 \div 9 = 63$ , $9 \div 7 = 63$ $9 \div 63 = 7$ , $7 \div 63 = 9$	15 (24%)	4 (10%)
$9 \div 7 =$ an answer	5 (8%) Answers: 3, 1r2, 9, 6, blank	6 (15%) Answers: 3, 6, 9, 2, blank, 1
$7 \div 9 =$ an answer	4 (6%) All wrote 1 r2	5 (13%) Blank, 23, 2, 1
Others:	13 (21%) Answers: $63 \div 7 = 6$ , $36 \div 7 = 9$ , $55 \div 9 = 54r1$ , $63 \div 9 = 54$ , $9 \div 6 =$ , $66 \div 9 =$ $6 \div 9 = 7$ , $1 \div 2 = 1$ , $9 \div 3 = 3$ $9 \div 1 = 9$ , $45 \div 5 = 9$ , $1 \div 7 =$	13 (33%) Answers: $\text{blank} \div 7 = \text{blank}$ , $63 \div 2 = 765$ $9 \div 6 = 1r3$ , $9 \div 9 = 7$ , $7 \div 63 = \text{blank}$ $6 \div 9 = 7$ , $7 \div 7 = 11$ , $36 \div 7 = 9$ $77 \div 11 = 7$ , $33 \div 9 = 7$ , $6 \div 7 = 9$ $93 \div 9 = 7$ , $1 \div 9 = 1$

Table 7: Incorrect responses to Question 1b by pupils who answered Question 1a correctly

#### Section 4: creating a context to match a symbolically represented calculation

Question 10 asked pupils to 'Write a problem for  $3 \times 7 = 21$ '. The research team did reflect afterwards that 'Write a story' or 'Write a context' may have required that the pupils make sense of and account for all three numbers in the calculation, whereas 'write a problem' does not require this.

Of the pupils who wrote a problem that matched the calculation, half used one of the same contexts from earlier in the paper and just under half used food as a context; there was an overlap between these as eggs were used as a context in the paper. There was no difference between the countries (Tables 8 and 9).

Analysis of correct responses to Q 10: Write a problem for $3 \times 7 = 21$		
Context chosen from earlier in the paper	Jersey	England
Chairs	8	15
Eggs	12	18
People/children/teams	14	19
<b>Total of contexts from previous questions</b>	<b>34/67 (50%)</b>	<b>52/102 (51%)</b>

Table 8: Contexts chosen from earlier in the paper for Question 10

Context type	Jersey	England
Fruit	3	9
Sweets/chocolate	8	7
Cakes/muffins/cookies/donuts/ baguettes	8	11
Sausages	1	0
<b>Total food</b>	<b>32/67 (48%)</b>	<b>45/102 (44%)</b>
Animals	5/67 (7%)	6/102 (6%)
Marbles/balls/balloons/bean bags/lego	3/67 (4%)	11/102 11%
Other	27/67 (41%)	40/102 (39%)

Table 9: Context types for Question 10

Around 40% of the correct responses from both countries were not able to be grouped; some reflected each pupil's personal experience (Table 10).

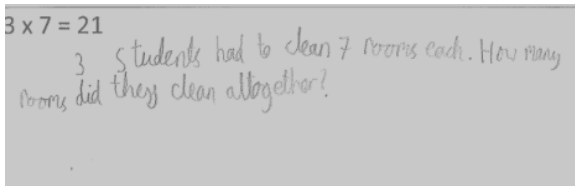
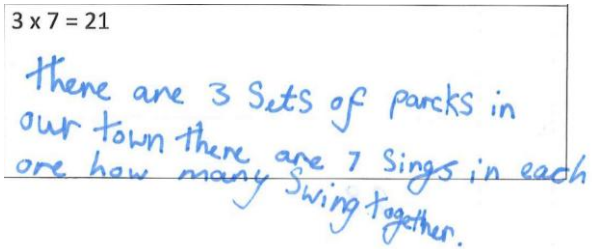
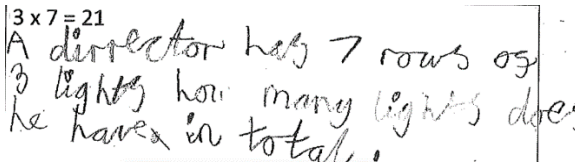
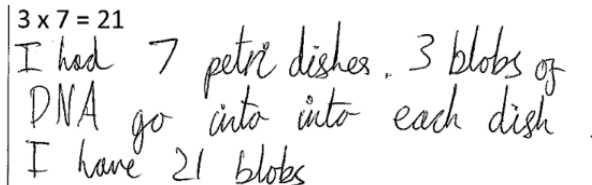
<p>Cleaning rooms</p> 	<p>Swings in park</p> 
<p>Lights in a theatre</p> 	<p>Blobs in petri dishes</p> 

Table 10: Examples of personal contexts used for Question 10

Most pupils wrote a multiplication problem (table 11):

	Multiplication	Division
England	83/102 (81%)	19/102 (19%)
Jersey	63/67 (94%)	4/67 (6%)

Table 11: Operation used in correct response problems for Question 10

In some multiplication problems the pupils used all three numbers

$$3 \times 7 = 21$$

there are 3 bags with  
7 sweets in each altogether  
there are 21 sweets

$$3 \times 7 = 21$$

The children are are arranged  
into 7 groups 3 children are in  
each group how many children are  
together? (21)

In most multiplication problems the pupils phrased the problem as a question, using only the numbers 7 and 3

$$3 \times 7 = 21$$

There are three boxes which  
each contain 7 apples - how  
many apples are there  
altogether?

$$3 \times 7 = 21$$

There are 3 rows of 7 seats  
how many seats are there?

Only 2 pupils in England and 3 pupils in Jersey used a scaling context, all the other pupils used repeated addition.

$$3 \times 7 = 21$$

Jessica has 7 eggs ~~on~~ on Friday. Jessica gets  
3 times the eggs on Sunday. How many eggs  
does Jessica have on Sunday?

$$3 \times 7 = 21$$

Bob bought 21  
apples and Fred  
got 3 times the amount  
of Bob's apples.  
How much apples did  
Fred buy?

A few pupils created a division problem, and all of these were phrased as a question:

$$3 \times 7 = 21$$

is there 3 groups of people and there's 21  
people how many people are in each group

$$3 \times 7 = 21$$

apples and 7 people.  
There were 21 bananas and 7 monkeys.  
you have how many apples did each  
person get?

Table 12: Examples of types of multiplication and division problems from Question 10

Further analysis of the multiplication and division problems responses (Table 12) showed that most multiplication contexts were repeated addition in some form, with only five that involved scaling. All of the scaling problems focused on discrete items; there were no problems involving measures. A small number of pupils (23 in total) chose to write a division problem.

## Data analysis: Teacher and Pupil Questionnaires

### Rationale of the questionnaires

The questionnaires were designed to be quick to complete and intended to uncover evidence of any impact the implementation of the MTC may have had on the teaching and wellbeing of pupils. We also hoped to learn about the perceptions of teachers and pupils regarding preferred strategies for teaching and learning multiplication facts.



After the pupils had completed the multiplication paper, they completed the short survey. Their class teachers were asked matching questions, including the same questions where appropriate.

As the data from the multiplication papers and questionnaires were anonymised it was not possible to cross reference pupil's responses to the questionnaire with their responses to the multiplication papers. Nor was it possible to link teachers' responses to those of their pupils. This placed limits on the analysis that could be used.

### Research questions

- Do teachers have a preferred approach when teaching multiplication facts?
- Do pupils have a preferred approach when learning multiplication facts?
- Are these preferred approaches similar?
- Has the MTC influenced the frequency of teaching and learning multiplication facts?
- Are attitudes to teaching and learning multiplication facts different between Jersey to England?

### Data Collection

There were 32 responses from teachers in Years 4 and 5 from England and 40 responses from the same year groups in Jersey. The pupil questionnaire was the same for pupils in Y4 and Y5. There were 912 responses from pupils in England and 1041 from pupils in Jersey.

### Choice of Questions

As with the multiplication questions the number and style of the questions were designed so that they would not be an imposition on teachers or onerous for pupils. Questions on attitudes towards multiplication facts and preferences on how to learn them were included.

### Findings from the teacher questionnaires

#### Question 1 was an open text box "What does it mean to be good at maths?"

Word frequency analysis was applied to the responses. Keywords were identified based on their relevance /frequency (Table 13)

Keyword	Jersey (n=40)	England (n=32)
Fluency	15%	31%
Problems	50%	47%
Solving	40%	44%
Reasoning	10%	13%
Knowledge	22%	22%
Understanding	15%	19%
Apply	30%	31%
Methods and Strategies	33%	16%

Table 13: Use of keywords by teachers in response to: What does it mean to be good at maths?

Although overall the similarities outweigh the differences, twice as many teachers referred to fluency in England than in Jersey where twice as many referred to method and strategy as being important.

### **Question 2: Knowing multiplications makes you good at maths**

Across both jurisdictions the majority of teachers, 65% believed it was important to know your times tables to be good at maths but a significant minority, 22%, disagreed with the statement. Teachers in Jersey were more unsure knowing your multiplication facts made you good at maths.

### **Question 3: You have to be fast to be good at maths**

The questionnaire revealed a tension between the beliefs as expressed by the teachers and their classroom practice with regard to speed in maths. The vast majority of teachers disagreed with the statement that you have to be fast to be good at maths, 86%. Of those teachers who agreed with the statement the majority came from England. However, in question 7, 69% of teachers in England and 58% of teachers in Jersey believed timed tests were a good way of learning multiplication facts presenting us with something of a paradox. This is against a research backdrop that suggests that timed tests may be less valid, less reliable, less inclusive and less equitable than untimed tests (Gernsbach et al, 2020), and that they may contribute to the onset of maths anxiety (Boaler, 2014).

### **Question 4: What level of anxiety have you seen in pupils when learning multiplication facts in the past year?**

Similar levels of anxiety around learning multiplication facts were shown across both jurisdictions with 75% of teachers highlighting anxiety shown in at least some of their pupils. None of the teachers who said they had no anxious pupils rated timed tests as a good way of learning multiplication facts.

### **Question 5: How many days in the week do you teach multiplication facts?**

38% of teachers in England responded that they taught multiplication facts everyday compared to 13% in Jersey and 66% of teachers in England identified teaching multiplication facts more than twice a week whilst this was only 51% of teachers in Jersey.

### **Question 6: Which resources do you use to help pupils to learn multiplication tables?**

In order of popularity across both jurisdictions, in relation to the resources listed on the questionnaire:

Arrays 89%, Multi table grid 88%, Number Line 60%, Place Value Counters 51% and Calculators 10%.

Number lines are shown greater preference in England, whereas place value counters are preferred in Jersey.

### **Question 7: What else do you use to help pupils to learn your multiplication tables?**

In order of popularity across both jurisdictions, in relation to the activities listed on the questionnaire:

Using a computer programme 89%, Games 88%, Answering multiplication questions 82%, Discussing how to find the answer 78%, Chanting/counting 67%, Timed test 63%

The data points to possible contradictions between stated beliefs of teacher about how best to learn mathematics from question 7 and the strategies they favour to teach multiplication facts.

89% of teachers use games/computer programs as a good way of learning tables but these can also be different versions of a timed test (the least popular option). Times Table Rock Stars for example claims to help pupils “recall their times tables in record speed”.

In addition, while using a computer aims to raise the engagement of pupils only 10% of teachers in Jersey and none in England considered using a calculator.

## Findings from the pupil questionnaires

The pupil survey had equivalent questions to the teacher survey except for question 1.

### Question 1 - This is how I feel about learning multiplication tables

There was a similar distribution across both jurisdictions on how pupils feel about learning their multiplication tables (Table 14)

Love it	Quite Like it	Don't mind	Don't like it much	Hate it
22%	25%	30%	15%	8%

*Table 14: Pupil responses to ‘This is how I feel about learning multiplication tables’*

We have considered the extremes, from the 22% who love learning multiplication facts, henceforth “the lovers”, to the 8% hating it (“the haters”). These apparent differences in pupils’ feelings about learning times tables then led to some interesting comparisons in subsequent questions. Unless specifically mentioned the differences in responses between England and Jersey were not significant.

### Question 2: Knowing multiplications makes you good at maths

While overall 51% agreed that knowing your tables makes you good at maths (lower than the teachers) this rose to 70% amongst “the lovers” and dropped to 25% of “the haters”.

### Question 3: You have to be fast to be good at maths

Overall, 82% pupils disagree that you have to be fast to be good at maths (slightly lower than the teachers). This was further backed by only 37% of pupils (Question 6) believing a timed test helped them learn their tables (cf 63% of teachers). However, the more they loved learning tables the more a timed test was preferred. This could quite possibly be influenced by the desire for performance goals.

### Question 4: How many days in the week do you teach multiplication facts?

Perception is everything.

When asked ‘How often do you teach/work on multiplication facts/tables’ teachers and pupils responded similarly in England and Jersey. However, when comparing the ‘lovers’ and ‘haters’ significant differences emerged (Table 15)

	Hardly ever	Once or twice a week	More than twice a week	Everyday
Teachers	8%	36%	33%	24%
Pupils	14%	36%	29%	21%
Lovers	8%	23%	30%	38%
Haters	45%	29%	10%	15%

*Table 15: Responses to 'How often do you teach/work on multiplication facts/tables?'*

Maybe “the lovers” of learning tables saw opportunities to use, or practise, them outside more formal classroom practice, and possibly at home, hence 38% work on tables every day, whilst “the haters” do not see many of the activities they work on as working on tables or do not seek additional opportunities to engage with learning tables.

#### **Question 5: Which resources do you use to help you to learn multiplication tables?**

	Table grid	Calculator	Number Line	Array	Place value counters
Teachers	88%	6%	60%	89%	51%
Pupils	51%	38%	28%	26%	30%
Lovers	50%	30%	28%	33%	31%
Haters	35%	54%	22%	18%	28%

*Table 16: Resources used for learning multiplication tables*

Teachers were more likely to identify resources that support the learning of tables than the pupils (Table 16). Only 50% of “the lovers” find a table grid helpful and 33% of these pupils find an array useful compared with 88% and 89% of teachers. These percentages for pupils drop further the more learning multiplication tables is disliked, yet for nearly all teachers these resources are believed to be helpful and heavily favoured. Clearly many pupils do not see the same value in these resources as their teachers.

Contrasting the preferences of “the lovers” and “the haters” we notice that the four resources that expose something about multiplicative structures (table grid, number line, array and place value counters) are favoured far more by the former than the latter. Is this because the “the lovers” can see the structure and so can make some sense of them, while “the haters” struggle to understand the relevance and function of them and consequently find them of less use?

For “the haters” the differences between their resource preferences and their teachers are greater and yet 54% identify a calculator being useful, the one resource that they are seemingly denied access to in class.

## Question 6: What else helps you to learn your multiplication tables?

There are clear differences between the pupil and teacher responses to what supports learning (Table 17) which raises the question ‘How much attention/ value do pupils give these activities?’

A greater proportion of pupils who hated learning facts found discussion more useful than chanting, answering questions or timed tests; potentially this helps them to make sense of the mathematics. “The lovers”, on the other hand, seem to have a preference for answering questions and timed tests. Carol Dweck (2000) suggested that a preference for getting questions right and looking smart were characteristics of a learner with performance goals, rather than learning goals.

	Games	Answering questions	Timed Test	Computer program	Chanting/counting	Discussion
Teachers	88%	82%	63%	89%	67%	78%
All Pupils	64%	40%	37%	43%	29%	37%
Lovers	59%	51%	56%	41%	26%	30%
Haters	64%	12%	13%	47%	20%	34%

*Table 17: Responses to ‘What else do you use to help pupils learn multiplication tables?’ and ‘What else helps you to learn your multiplication tables?’*

Questionnaires summary:

- There is little evidence of different attitudes to learning tables across England and Jersey. Teachers in England reported working on tables somewhat more often than in Jersey.
- Universally speed was not considered important in learning tables yet significantly timed tests and computer games were seen by teachers, and some pupils, as positive ways to learn tables.
- In the apparent enjoyment of learning tables, “the lovers” and “the haters” showed great variance into how often they believed they worked on times tables and what was useful to them when they did
- Constructs such as number grids and arrays are not seen as useful by pupils as they are by teachers. Greater emphasis may be needed on their multiplicative structure.
- Proportionally discussion is valued more by pupils identifying as hating learning their multiplication tables as opposed to those who love learning them.
- Anxiety is a significant factor when learning multiplication facts.

## Discussion

Whilst the catalyst for this research was the MTC, used in England but not Jersey, the results from both the mathematics papers and the attitude questionnaires tell us about much more, providing an insight into how pupils and teachers think about multiplicative reasoning and prompting questions about how multiplicative relationships are understood and taught.

Overall, performance across all 10 mathematics questions was poor. The highest performing question for pupils in both jurisdictions was Question 6, where close to 70% of pupils achieved a correct response. However, this also reveals that almost one third of the pupils could still not answer the 'easiest' question on the paper.

The deliberate decision to make the necessary multiplications the same in Q3 and Q6, allows a distinction to be made between recognising and using and applying. Q6 involved recognising a multiplication in a symbolic form, as it would be presented in the MTC and would have been seen many times, whereas Q3 involved making sense of a context and 'recognising' it as one that required the same multiplication, in other words using and applying the multiplication. There is a difference between performance on these two questions, with performance on Q3 significantly lower than Q6 in both countries. The comparison of Q3 and Q6 also gives some indication that pupils in Jersey were better at recognising when to use a multiplication they know in a context. This prompts questions about whether pupils understand when the multiplication bonds they have learnt might be used and whether some are relying on recognising, i.e. they only expecting to use learnt multiplication bonds when given a written multiplication, such as  $11 \times 12$ .

The difference between recognising and producing also seems to be evident in these results (Dehaene 2020, Coles 2024). Q1 asked the pupils to 'produce' a multiplication and a division to match an image. The number of pupils who could write a correct multiplication was low in England and Jersey, only just over half, and even lower for a division, suggesting the pupils did not have a secure understanding of the relationship between multiplication and division.

Often, children left a blank response for the division or did not use the same three numbers as they had used in the multiplication. These observations provoke questions about the teaching of multiplicative reasoning. At the heart of multiplicative reasoning is understanding the relationship between multiplication and division, yet the evidence here suggests that multiplication and division may be being taught in isolation from each other, with more time and attention given to the teaching of multiplication (the focus of the MTC).

If teachers explore multiplication and division simultaneously, in different contexts, pupils are required to make sense of each of the numbers in a related trio. Coles (2024) suggests 'Inverse processes gain meaning from each other...seek a representation that allows for doing and undoing'.

Producing rather than recognising was also a significant component of Q10 where pupils were asked to produce a context to match a given multiplication; this had a very low success rate. This again prompts a question about the focus of teaching multiplicative reasoning and whether it is on recognising written multiplications and responding to these, with limited opportunities to make sense of the multiplications and divisions in context including limited opportunities to produce contexts which demonstrate an understanding of the multiplicative relationship.

The questionnaires provide some insight into the thinking of teachers with regard to teaching multiplicative reasoning. There is evidence that when people are feeling stressed by the time pressure of a test, their working memory is negatively affected (Lyons and Beilock 2011, Beilock

and Willingham 2014). The English teachers have no control over the fact that the MTC is a statutory **timed** assessment, however they can choose how to teach children about multiplicative relationships including multiplication bonds. This provokes a question about why nearly  $\frac{2}{3}$  of the teachers in this study believed that timed tests help children learn the bonds and might choose to use them for this learning. Is an unintended side effect of the MTC that some teachers believe that giving children timed tests is ‘teaching’ multiplication bonds?

We also found that pupils who love learning their multiplication bonds are four times as likely to love timed tests. It maybe that this group, who may appear ‘successful’ in the maths classroom, influences teacher decisions. How can teachers account for all learners, when learning multiplication bonds, to ensure that increased anxiety and a decreased love of maths are not unforeseen consequences?

One of the biggest disjunctions we found through the questionnaires was the difference between which resources the teachers thought helped their pupils to learn multiplication bonds and what their pupils thought (see Table 16). This was most pronounced with the array. Many mathematics teachers and researchers believe that the array is a powerful tool for representing multiplicative relationships that can help learners understand multiplication and division concepts (Jacob and Mulligan, 2014; Barmby and Harries, 2007), but what does it mean if the learners can’t see this?

One possibility could be that pupils don’t understand the model. Outhred and Mitchelmore (2004) reported difficulties with learners accessing the meaning of the array including a failure to understand, or be able to represent, rectangular arrays. Many of the learners in their study failed to recognise the essential regularity of the array. They go on to observe that:

*Although it may seem self-evident to adults that the number of units in the array must depend on the measurements of the sides, it was clearly not obvious to students (p.471).*

Kucheman and Hodgen (2018) also recognised that teachers can overlook the struggles learners may encounter when engaging with the array.

*We have argued that the rectangular array and, in turn, the area model, provide powerful models of multiplication that can help students analyse the structure of multiplication. However, this facility is hard won. We tend to underestimate the difficulties that students encounter in getting to grips with the array and the area model (p.14).*

So, although the teachers in our study could clearly see the potential for usefully employing the array, it may well be the case that their learners could not make sense of the array and that the teachers did not recognise this.

Finally, there was a significant difference between those pupils who love learning their multiplication bonds and those who hate this, in both how often each group believes they work on learning their tables and which activities help them to learn. This suggests that different pupils may or may not be focussing on multiplication bonds during the same activity; for example, engagement in a computer game does not ensure that the pupil gains the conceptual understanding to intellectually engage with the mathematics. As Harel (2008) suggests:

*For students to learn the mathematics we intend to teach them, they must have a need for it, where ‘need’ refers to intellectual need, not social or economic need. (p.20)*



### **Considerations for teachers, researchers and policy makers arising from the study:**

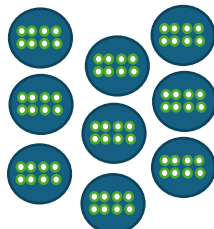
- Re-define fluency so that it includes flexible thinking and making connections across mathematical facts.
- Teach multiplication bonds as part of multiplicative reasoning, focusing on the multiplicative relationship between trios of numbers (for example 3, 4 and 12). This would support multiplication and division being taught together.
- Focus on developing understanding of structures within multiplicative reasoning and increase related teacher professional development.
- Support pupils to use and apply their knowledge in different contexts and include contexts with excess information that demand pupils think deeply and have to make sense of the mathematics in the context.
- Provide opportunities for pupils to 'produce' representations of multiplicative relationships and not just 'recognise' symbolic multiplications and divisions. This includes producing calculations to match structured mathematical images, contexts and words, and producing contexts, images and language to match symbols, different representations identified in the Connective Model (Trundle et al 2024).
- Provide guidance for primary schools which outlines multiplication bonds for each year group, so that deriving bonds builds on existing knowledge, and related professional development exploring specific manipulatives and approaches for supporting learning multiplication bonds.
- Avoid timed testing as part of retrieval practice due to the negative impact on pupils who experience anxiety and remove high stake accountability measures attached to multiplication tables, so that multiplication tables can be taught without the pressure and anxiety of timed testing.
- Future research might be directed towards the question of the teacher's perception of the learner's understanding of representations. In particular, looking at representations whose structure may seem 'self-evident' to the teacher.



## Appendix 1: Year 4 Maths Questions and illustrations

Practice:  $50 + 50$

1. Write down a multiplication fact and a division fact represented by this diagram, which is on your sheet.



2. There are five rows of nine chairs laid out in the school hall for a parents evening. How many chairs are there altogether?
3. At lunchtime in the hall six children sit down at each table. Ten children have packed lunch. There are eight tables. How many children sit down altogether?
4. Eggs are packed in boxes of 6. The farmer collects 56 eggs. How many boxes can they fill?

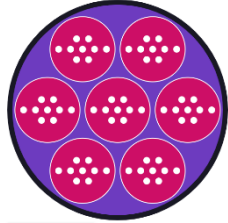


5.  $4 \times 7$
6.  $8 \times 6$
7.  $72 \div 9$
8.  $19 \times 3$
9.  $280 \div 7$
10. Write a problem for  $3 \times 7 = 21$

## Appendix 2: Year 5 Maths Questions and illustrations

Practice:  $50 + 50$

1. Write down a multiplication fact and a division fact represented by this diagram, which is on your sheet.



2. There are eight rows of nine chairs laid out in the school hall for a parents evening. How many chairs are there altogether?
3. Teams of twelve are needed for a competition. Four teachers arrange the children into eleven teams. How many children are there in total?
4. Eggs are packed in boxes of 12. The farmer collects 114 eggs. How many boxes can they



fill?

5.  $8 \times 7$
6.  $12 \times 11$
7.  $63 \div 9$
8.  $6 \times 19$
9.  $222 \div 3$
10. Write a problem for  $3 \times 7 = 21$

## Appendix 3: Pupil Questionnaire

### Question 1

This is how I feel about learning multiplication tables

Love it	Quite like it	Don't mind	Don't like it much	Hate it
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### Question 2

Knowing multiplications makes you good at maths

Agree	Not sure	Disagree
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### Question 3

You have to be fast to be good at maths

Agree	Not sure	Disagree
-------	----------	----------

### Question 4

How often do you work on multiplication tables?

- Hardly ever
- Once or twice a week
- More than twice a week
- Everyday

### Question 5

Which resources help you to learn your multiplication tables?

- Multiplication table grid
- Calculator
- Number line
- Arrays
- Place value counters
- Numicon
- Another resource: please say which one

### Question 6

What else helps you to learn your multiplication tables?

- Games
- Answering multiplication questions
- Timed test
- Using a computer programme
- Chanting/counting
- Discussing how to find the answer
- Something else

## Appendix 4: Teacher Questionnaire

### Question 1

What does it mean to be good at maths?

### Question 2

Knowing multiplication facts makes you good at maths

Agree	Not sure	Disagree
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Other comments:

### Question 3

You have to be fast to be good at maths

Agree	Not sure	Disagree
-------	----------	----------

Other comments:

### Question 4

What level of anxiety have you see in children when learning multiplication facts in the past year?

All children are anxious	Many anxious children	Some anxious children	A few anxious children	No anxious children
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Other comments

### Question 5

How many days in the week do you teach multiplication facts?

- Hardly ever
- Once or twice a week
- More than twice a week
- Everyday

England only: Has this changed in response to the MTC? How?

### Question 6

Which resources do you do you use to help pupils to learn multiplication tables?

- Multiplication table grid
- Calculator
- Number line
- Arrays
- Place value counters
- Numicon
- Another resource: please say which one

Which resource do you think works best? Why?

### Question 7

What else do you use to help pupils learn multiplication tables?

- Games
- Answering multiplication questions
- Timed test
- Using a computer programme
- Chanting/counting
- Discussing how to find the answer
- Something else

Which approach do you think works best? Why?

### Question 8

ENGLAND only: Has the MTC had an impact on your teaching of mathematics?

Very high impact	High impact	Some impact	A little impact	No impact
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How has the MTC had an impact on your teaching of mathematics?

JERSEY only: If you had the opportunity use the MTC would you?

- Yes
- Not sure
- Don't mind either way
- No

Why?

## Appendix 5: Pupil Sample Data Coding

### Y5 questions:

#### Question 1:

- Blank
- $9 \times 7 = 63$
- $7 \times 9 = 63$
- Other multiplication
- $63 \div 9 = 7$
- $63 \div 7 = 9$
- Other division

#### Question 2:

- Blank
- Correct 72
- Incorrect
- Correct calc. no answer
- Counted

#### Recordings:

- Drawing
- " $8 \times 9 = 72$  or  $9 \times 8 = 72$  or  $9 \times 5 = 45$ "
- Other calc
- Comment

#### Question 3:

- Blank
- Correct 132
- Incorrect
- Correct calc. no answer
- Counted

#### Recordings:

- Drawing
- $11 \times 12$  or  $12 \times 11$
- Other calc
- Used 4 somewhere

Question 4:

- Blank
- 9
- 9 r 6

Recordings:

- Written method
- Drawing
- $114 \div 12$

Question 5:

- Blank
- 56
- Incorrect
- Counted

Recordings:

- Drawings
- Other

Question 6:

- Blank
- 132
- Incorrect
- Counted

Recordings:

- Drawings
- Other

Question 7:

- Blank
- 7
- Incorrect
- Counted

Recordings:

- Draw grouping
- Draw sharing
- Other

Question 8:

- Blank
- 114
- Incorrect
- Counted

Partitioning:

- 10 and 9
- 20 and -1

Recordings:

- Written method
- Drawing
- Other
- Comment

Question 9:

- Blank
- 74
- Incorrect
- Counted
- Links to x

Recordings:

- Written method
- Drawing
- Other
- Comment

Question 10:

- Blank
- Correct x
- Correct  $\div$
- Adds 3 and 7
- Uses the numbers but not connected to the calculation
- Writes a related fact
- Comments on the calculation
- Used a context from earlier in the questions
- Other



## References

- Anghileri, J., Beishuizen, M., & Van Putten, K. (2002) From informal strategies to structured procedures: Mind the gap! *Educational Studies in Mathematics*, 49 (2), 149-170.
- Association of Teachers of Mathematics and the Mathematical Association (2001) The Teaching and Learning of Multiplication Bonds: a position Statement.  
<https://www.m-a.org.uk/resources/1multiplicationbondsATMMA.pdf>
- Barmby, P. and Harries, T. (2007) Representing and understanding multiplication *Research in Mathematics Education* 9 (1). pp. 33-46.
- Barmby, P. Harries, T. Higgins, S. E. and Suggate, J. (2009) The array representation and primary children's understanding and reasoning in multiplication *Educational Studies in Mathematics* 70(3):217-241.
- Beilock S L and Willingham D T Ask the Cognitive Scientist: Maths Anxiety, Can Teachers Help Students Reduce It? *American Educator*, 28-32.
- Boaler, J. (2014) Fluency without Fear [Fluency Without Fear - YouCubed](#)
- Borthwick, A & Harcourt-Heath, M. (2016) Calculation Strategies for Year 5 children: 10 years on. *Proceedings of the British Society for Research into Learning Mathematics* 36 (3).
- Coles, A. (2024) *Social justice, cognitive science and the powers of learners* MT294 ATM.
- Department for Education (DfE) (2013) *Mathematics programmes of study: Key Stage 1 and 2. National Curriculum in England*. London: UK: DfE.
- Dweck, C. S. (2000). *Self-theories: Their role in motivation, personality, and development*. Philadelphia, PA: Psychology Press.
- Foster, C. (2014) Mathematical Fluency Without Drill and Practice. *Mathematics Teaching*, 240. ATM.
- Gattegno, C. (2010). *What we owe children: The subordination of teaching to learning*. Educational Solutions (first published 1987).
- Gernsbacher M. A., Soicher R. N., & Becker-Blease, K. A. (2020). Four Empirically Based Reasons Not to Administer Time-Limited Tests. *Translational Issues in Psychological Science*, 6(2): 175–190. <https://doi.org/10.1037/tps0000232>.
- Government of Jersey (2018). *Jersey Curriculum – Mathematics*. Government of Jersey.
- Harel, G. (2008). *What is mathematics? A pedagogical answer to a philosophical question*. In B. Gold & R. Simons (Eds.), *Proof and other dilemmas: Mathematics and philosophy* (pp. 265–290). Washington, DC: Mathematical Association of America.
- Jacob, L., & Mulligan, J. (2014). Using arrays to build towards multiplicative thinking in the early years. *Australian primary mathematics classroom*, 19(1), 35-40.
- Kucheman, D. and Hodgen, J. (2018) Using the array model to develop multiplicative reasoning *Mathematics Teaching* 261, 11-14.
- Lyons, I M and Beilock S L (2011) Mathematics anxiety: separating the maths from the anxiety, *Cerebral Cortex*.
- Outhred, L. and Mitchelmore, M. (2004) Students' Structuring of Rectangular Arrays *Proceedings of the 28th Conference of the International Group for the Psychology of Mathematics Education*, 2004 Vol 3 pp 465–472.
- Trundley, R., Tynemouth, A., Edginton, H. & Burke, S. (2024) How Haylock and Cockburn and the connective model have shaped and inspired our thinking for 25 years MT293 ATM.  
<https://atm.org.uk/write/MediaUploads/Journals/MT293/06.pdf>